

Feedback Rules, Structural Endogeneity and the Lucas Critique in a Small Macro Model

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Abstract

This paper presents an alternative approach in testing for the Lucas critique, using the Kalman filter, within the context of a small open economy that follows inflation (exchange rate) targeting rules. A stylized model was estimated and time-varying coefficients were used to test for the relationship between policy-regime changes and model coefficient stability for the Dominican Republic. The results support the conclusions of the Lucas critique, which has important implication for the performance of macroeconometric models in general and for the evaluation of macroeconomic policy in particular. The method also provides a way to partially overcome Lucas critique implications.

JEL Classification: E5; F3

Keyword(s): Lucas critique, recursive estimation, feedback rules, small open economy, macroeconometric models.

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1. Introduction

In his seminal article, Lucas (1973, p.126) stated the following critique:

"Given that the structure of an econometric model consists of optimal decision rules of economic agents, and that optimal decision rules vary systematically with changes in the structure of series relevant to the decision maker, it follows that any changes in policy will systematically alter the structure of econometric models."

Econometric models are systems of mathematical equations that describe the relationship among stochastic economic variables such as output, inflation and exchange rates, and their interaction with policy variables such as government spending and monetary regimes. These tools were extensively used for economic forecasting and simulation, and taken on as a standard policy assessment instrument in many developed and developing countries. Ever since the Lucas critique, however, the popularity of econometric models tumbled provided that theoretically, the critique implies that policy simulations modify the *ex-post* structural and statistical relationship of the system, making simulations inconsistent and biased.

Lucas (1975 and 1977), and Kindal and Prescott (1982) among others, argue that models constructed under rational expectations make simulations immune to the critique. There is evidence, however, that backward-looking models perform statistically better than forward-looking alternatives, rising doubts on the validity of the rational expectation argument (see Estrella and Fuher, 1999). In general, the critique does not depend entirely on rational expectations, but on the existence of active endogenous policy rules.

There has been an extensive and yet inconclusive debate on whether the critique holds empirically. One of the leading assessment tools is the concept of super exogeneity among stochastic variables (see Sims, 1982; Engle et. al, 1983;

Engle and Hendry, 1993) although there is also evidence that supply exogeneity is not capable of detecting it within small samples (see Lendé, 2000).

This paper presents an alternative approach in testing for the critique using time-varying coefficient estimation within the context of a small open economy with rational expectations that follows inflation (exchange rate) target rules in the conduct of monetary policy. The time-varying coefficients of the structural relationships in the econometric model are used to test for stability and are simultaneously related, using the Kalman filter, to policy feedback rules. This technique, related to the one used by Agenor and Taylor (1992) within context of structural parameter changes during stabilization, allows testing for the critique as well as providing an endogenous method for overcoming its implications. The method is applied to the particular case of the Dominican Republic (DR), which is of particular interest given its historical records of different political and economic regimes, including several stabilization programs introduced during the seventies, eighties and nineties.

In what follows, section 2 presents a small open economy rational expectations macroeconomic model with policy feedback rules, and shows how “deep” structural parameters, in Lucas (1973) sense, are related to policy feedback preferences. Section 3 estimates the model for the DR and runs simulation tests for the critique using time-varying coefficient techniques. Section 4 concludes.

2. Feedback rules and the Lucas critique

As a way of visualizing the workings of the critique and to develop a testable framework, a stylized model, presented by equations (1)-(4), is developed.¹

¹ This model has been tailor made for the Dominican Republic working backwards from an empirical estimate of the money rule towards the supply function. Once the structure of the money rule is known and how expectations affect the supply equation, the structure suggested by equation (1)-(3) is easily developed.

$$y_{t+1} = \bar{y} + \mathbf{a}[p_{t+1} - E_t p_{t+1}] + \mathbf{b}y_t + \mathbf{m}_{t+1}, \quad (1)$$

$$m_{t+1} - p_{t+1} = \mathbf{d}y_{t+1} - \mathbf{g}\Delta E_t p_{t+1} + \mathbf{n}_{t+1}, \quad (2)$$

$$\Delta m_{t+1} = \mathbf{I}\Delta E_t e_{t+1} + \mathbf{e}_{t+1}. \quad (3)$$

$$\Delta E_t e_{t+1} = \Delta E_t p_{t+1} \quad (4)$$

Equation (1) is a Lucas type aggregate supply with technological drift (see Lucas, 1972), where y is the log of real output, p is the log of domestic prices, \bar{y} is the log of potential output, E is a conditional expectations operator, and \mathbf{m} is a random supply shock.² Equation (2) is aggregate demand, where m is the log of M1, Δ is a difference operator, Δp is used as a proxy of the cost of holding money (see Agénor and Khan, 1996; and Carruth and Sánchez-Fung, 2000), and v is a random velocity shock. Equation (3) is a money rule in which the monetary authorities imperfectly adjust money balances proportionally, by a factor \mathbf{I} , to expected changes in the exchange rate e_t subject to a random measurement error \mathbf{e} .³ Finally, equation (4) states that PPP holds.⁴

In the standard framework, the authorities would choose to minimize a loss function in the variance of output and inflation by selecting the appropriate value of \mathbf{I} subject to a reduced form of equation (1). In order to find that reduced form, however, the procedure is to solve for unknown expectation. Substituting (1) and

² Hereafter, all random shocks in the system are assumed i.i.d with zero mean and finite variance.

³ In equation (3), λ measures policy preferences. Values of λ close to unity indicate an accommodative monetary policy, whereas values close to zero show a restrictive monetary policy. Without loss of generality, the authorities have complete control over λ .

⁴ Foreign prices are assumed constant.

(3) into (2) and taking conditional expectations, results in the following expected price solution

$$E_t p_{t+1} = \frac{1}{1-g-l} [m_t - d\bar{y} - db y_t - (g+l)p_t]. \quad (5)$$

Equation (5) is the standard rational expectation result with policy neutrality.⁵ Substituting (5) into (1) and collecting terms, gives the following quasi-reduced error correction supply curve

$$\Delta y_{t+1} = s_1 + s_2 \Delta p_{t+1} + s_3 p_t + s_4 m_t + s_5 y_t + \mathbf{m}_{t+1}. \quad (6)$$

Equation (5) is a standard solution with structural coefficients defined by

$$s_1 = \bar{y} \left[1 + \frac{ad}{1-g-l} \right], \quad (6.1)$$

$$s_2 = \mathbf{a}, \quad (6.2)$$

$$s_3 = \mathbf{a} \left[1 + \frac{g+l}{1-g-l} \right], \quad (6.3)$$

$$s_4 = \frac{-\mathbf{a}}{1-g-l}, \quad (6.4)$$

$$s_5 = \mathbf{b} \left[1 + \frac{ad}{1+g-l} \right] - 1. \quad (6.5)$$

⁵ Note that neutrality is due to anticipation of policy changes both in the variables and the underlying parameter of the model (see Sargent and Wallace, 1976).

Note that each coefficient depends on the “deep” structural parameters of the base model, as well as on the rule or policy regime the authorities choose to follow (i.e., \mathbf{I}). As a consequence, changes in \mathbf{I} will shift all structural coefficients in (5), making any policy simulation incompatible among alternative scenarios.⁶ However, the fact that the structure of the underlying coefficients is known allows to empirically endogenise the coefficients as to account for such regime changes.

3. Empirical evidence

Equation (6) and the money feedback rule, equation (3), were estimated for the DR for the period between 1965 and 1999 using annual data for real GDP, inflation (measured by the percentage changes in domestic CPI), and real M1. Data source and description are presented at the appendix.

The estimation process is as follows. First, the statistical properties of the variables are evaluated for stationarity. Second, a feedback rule is estimated using time-varying coefficient techniques. Finally, equation (6) is estimated using the Kalman filter while nesting the state coefficients to the time-varying feedback coefficient obtained from the money rule.

Stationarity of the individual series is presented in table 1 using the Augmented Dickey-Fuller (ADF) test with a three lags truncation. The results show that the log of real GDP, the log of M1, and the log of CPI follow $I(1)$ processes, providing a consistent set of first differences covariance stationary Gaussian processes for the structure proposed by equation (6).

Table 2 presents the IV estimation for the money rule.⁷ The results show, with high significance, that the Central Bank adjusts money supply in proportion to

⁶ The fact that all coefficients are related to \mathbf{I} is the result of assuming a multiplicative policy feedback rule.

⁷ The instruments used where the constant, exogenous and lagged endogenous variables namely m_{t-1} , p_{t-1} , y_{t-1} . A fourth variable was needed to identify the supply equation. The obvious candidate was the recursive measure of λ_t obtained from the money rule estimate.

the expected level of inflation. The average reaction is of about 36% ($\lambda = 0.36$) with substantial variation from periods of restrictive to accommodative monetary policy.⁸ Figure 3 indicates that the Central Bank followed a loose monetary policy during the seventies, coincidentally when the Dominican peso was pegged to the US dollar. On the other hand, when the peg was lost at the beginning of the eighties, the Central Bank appeared to simultaneously tighten its policy stance allowing lower values of the feedback coefficient. The control over the peg was regained and again lost in 1985, apparently due to the speculative attacks on the peso, causing the worst devaluations in the history of the Dominican exchange rate market. In general, if the critique holds empirically, this variability should be reflected in the structural stability of the coefficients of equation (6).⁹

Table 3, shows equation (6) estimated under the traditional (constant coefficients) approach.¹⁰ In general, real output appears to be co integrated with prices and money, with a normalized vector given by

$$m - p = 6.91 + y \quad \forall t$$

The above-normalized vector implies a long-run money demand equation consistent with Friedman's (1962) quantity theory assumptions.¹¹ Turning to short-term dynamics, the coefficient of inflation indicates that the elasticity of output to expected inflation is high and about 0.87, suggesting a strong response of output to

⁸ Several estimation methods (i.e., Kalman filters), as well as several specifications involving different lag structures and different explanatory variables, such as exchange rates and output, were tested. The results presented are the most sensible.

⁹ An ADF test [-4.34, {-1.95}] confirmed the stationarity of the errors at the 5% critical level.

¹⁰ Note that OLS is a special case of a Kalman filter under the assumption of constant coefficients.

¹¹ The results imply that the supply equation is in fact the money demand in the steady state and neutral according to its empirical coefficients.

expected changes in prices.¹² Regarding diagnostics, the log-likelihood is about -31.68, and the AIC is 2.096. Finally, the ADF test on the residuals (-4.34, {-3.75}) suggests that supply shocks are stationary at the 1% critical level (see figure 2).¹³ Regarding coefficient stability, figures 4 through 8, show the recursive estimates of the coefficients of equation (6). The results show substantial variability in all of the coefficients especially, during the mid eighties, around the time of the first peso problem and inflationary rally in the RD.¹⁴

Turning to the Lucas critique, the test implies nesting the state coefficients of equation (6) to the time-varying feedback coefficient of equation (3) in order to test for a significant relationship. The results will be compared to the constant coefficient case using the regression statistics and the Akaike Information Criterion (AIC). Table 4 shows the results for a nested state space model that uses the time-varying feedback coefficient from the money rule as controls in the state equations. The estimation is carried out using the Kalman filter with a diagonal covariance matrix for both the signal and state equations. The coefficient for inertial inflation was allowed to follow a random walk process while the coefficient g was estimated by the filter.¹⁵ The findings are noteworthy. All of the coefficients in the state space model, except the constant terms, were significant at the 1% level. The magnitudes and signs of the final state coefficients are consistent with those of table 3, and the log-likelihood (51.73) indicates a substantial improvement

¹² The reason why these elasticities are negative is due to the use of real GDP instead of nominal output in the supply equation. As the log of the price level is subtracted from the log of nominal output in order to convert to real quantities, unity must also be subtracted from the coefficient a of the price level in the equation. This suggests that the underlying value of a for the DR is about 0.75.

¹³ Idem footnote 5.

¹⁴ The n -step-ahead recursive Chow test (not shown) showed significant outlying probabilities in 1982 ($p > 0.1$), 1983 ($p > 0.1$), and 1984 ($p > 0.05$).

¹⁵ Several theories imply that inertia inflation is directly related to credibility that in turn depends on policy regimes and other policy variables (see Edwards, 1998; and Agénor and Taylor, 1992). However, in order to have an explicit structure for inertia, a model for agents' preferences is required.

of the predictive power of the model after structural endogeneity is accounted for. The AIC of -2.49 also favors the structurally endogenous model over the traditional approach. The results have interesting implications; however, the most consequential is that policy conduct as measured by the money rule coefficient I , has a significant “deep” impact on all of the structural coefficients of the supply equation.

As a way of illustrating the magnitude of the biased incur if structural Endogeneity is overlooked, table 5 shows simulations using the Gauss-Seidel (see Hamilton, 1994) algorithm for a model that includes an aggregate supply equation, a money rule and a long run aggregate demand equation. The simulations compare three different scenarios within the period between 1995 and 1999. The first is a base simulation assuming that the coefficients of the supply equation remaining at their constant regression values. The second simulation assumes a one time 5% increase in the feedback coefficient I , maintaining the state equations at their base line simulation values. Finally, simulation 3 incorporates the effects of the feedback coefficient in both the money rule and the state equations. The exercise attempts to capture the net effect of a change in the monetary policy reaction over the traditional model and the structurally endogenous case. The difference between the last two simulations should isolate the Lucas critique contribution (see figures 9 and 10).

The dynamic multiplier for a 5% increase in the feedback rule shows a 1% falls in the first year and half of a percentage point in the second year of the base case simulation. The increase in subsequent periods is from 1% up to 1.4% percent. The multiplier for second simulation shows the same pattern with magnitudes ranging from 2% to 3.8%. The net effect, however, shows that the Lucas critique accounts for about 2.7% of the dynamic multipliers in the first period, 1.3% in the second, and from 2% to 2.4% in the subsequent periods. This implies that the impact of policy changes on the structure of the system has more

weight than the direct effect of policy on the system itself. In this regard, omitting structural endogeneity causes a significant double bias in the forecast of the traditional model, that lie outside the usual confidence bands (see figure 10).

As it appears, structural endogeneity changes the underlying magnitude of the fundamentals of the economy, which in turns affects the choice of optimal paths and instruments in the conduct of policy. These findings suggest that the Lucas critique appears to hold significantly for the DR case.

4. CONCLUDING REMARKS

This paper tested the Lucas critique in the context of a small open economy that follows inflation (exchange rate) target rules in the conduct of monetary policy. The results indicate that the choice of policy significantly affect all of the coefficients in a reduced form system, enlightening the empirical relevance of the critique. Simulation performed with the model show that the bias produced by the Lucas critique are about two times higher than the standard model that ignores structural change. The time varying coefficients of a stylized model are used to test for stability and for regime-coefficient dependence. The technique is tested on the particular case of the Dominican Republic. The results indicate that the choice of policy significantly affect all of the coefficients in a reduced form system, enlightening the empirical relevance of the critique. Simulation performed with the model show that the bias produced by the Lucas critique are about two times higher than the standard model that ignores structural change. The method also provides a way to incorporate structural changes and to partially overcome the implications of the critique.

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Table 1: Phillips-Perron UR test statistic

Variable	PPT (trend and constant)
$\ln y_t$	-1.747
$\Delta \ln y_t$	-6.730*
$\ln M1_t$	-1.018
$\Delta \ln M1_t$	-6.757*
$\ln P_t$	-1.557
$\Delta \ln P_t$	-3.545**

Note: * and ** denotes significance at the 1% and 5% level under MacKinnon critical surface. All estimations using Eviews 3.4. The data is available by request to the authors.

Table 2: regression results for a money rule (1950-1999)

$$\Delta^* m_t = \underset{(0.175)}{0.36} \Delta p_t$$

$$R^2 = 0.994, DW = 2.03, N = 25, SE = 0.11, F = 1920.55 (p < 0.01)$$

Note: standard errors in parenthesis

*A Wald coefficient restriction test for an ADL (1,1) in the money rule, rejected the null that the coefficient of the lag dependent variable m_{t-1} was different than unity [$\chi^2 = 13.75$ ($p < 0.01$)]. It appears that both representations are robust.

Table 3: State Space model with constant coefficients for GDP (1950-1999)

	Coefficient	Standard Error	z-Statistic
c(1)	-0.952	0.007	-144.616
c(2)	-0.127	0.015	-8.516
c(3)	0.149	0.002	76.696
c(4)	-0.149	0.001	-106.533
c(5)	0.150	0.001	224.69
	Final State	Root MSE	z-Statistic
s1	-0.952	<0.001	–
s2	-0.127	<0.001	–
s3	0.149	<0.001	–
s4	-0.149	<0.001	–
s5	0.150	<0.001	–
Log-likelihood	-31.686	AIF	2.096
Parameters	5	ADF	-4.34
Diffuse Priors	0	Adjusted N	35

$$\Delta y_t = s1_t + s2_t \Delta p_t + s3_t p_{t-1} + s4_t m_{t-1} + s5_t y_{t-1}$$

$$s1_t = c(1) \quad \forall t$$

$$s2_t = c(2) \quad \forall t$$

$$s3_t = c(3) \quad \forall t$$

$$s4_t = c(4) \quad \forall t$$

$$s5_t = c(5) \quad \forall t$$

Note: Diagonal convenience for state and signal equations.

Table 4: State Space model with controlled states for GDP (1950-1999)

	Coefficient	Standard Error	z-Statistic
c(1)	-7.615	0.319	-23.804
c(11)	0.035	0.022	1.639
c(12)	0.252	0.003	7.878
c(31)	-0.003	<0.001	-11.498
c(32)	0.003	<0.001	-11.889
c(41)	0.064	0.001	5.303
c(51)	-0.039	0.001	-1.373
c(52)	-0.038	0.005	-7.873
	Final State	Root MSE	z-Statistic
s1	0.835	0.065	12.910
s2	-0.188	0.025	-7.444
s3	-0.123	0.017	-7.444
s4	0.123	0.017	7.444
s5	-0.112	0.009	-11.413
Log-likelihood	60.477	AIF	-2.942
Parameters	9	ADF	-6.420
Diffuse Priors	5	Adjusted N	35

$$\Delta y_t = s1_t + s2_t \Delta p_t + s3_t p_{t-1} + s4_t m_{t-1} + s5_t y_{t-1}$$

$$s1_t = c(11) + c(12)[s2_{t-1}/(1 - I_t - \mathbf{g})]$$

$$s2 = s2_{t-1}$$

$$s3_t = c(31)s2_{t-1} + c(32)[s2_{t-1}(I_t + \mathbf{g})/(1 - I_t - \mathbf{g})]$$

$$s4_t = c(42)[s2_{t-1}/(1 - I_t - \mathbf{g})]$$

$$s5 = c(51) + c(52)[s2_{t-1}/(1 - I_t - \mathbf{g})]$$

Table 4: Simulations and Dynamic Multiplier

	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999
Base case forecast with controlled state space model	4290.3	4515.9	4723.6	4885.6	5030.1	5179.8	5309.8	5402.3	5373.5	5361.9
Simulation 1: 5% Increase in feed back coefficient with no effect of state equations	4290.3	4515.9	4723.6	4885.6	5030.1	5124.2	5275.9	5352.3	5311.8	5288.4
Dynamic multipliers for simulation 1	0.0%	0.0%	0.0%	0.0%	0.0%	-1.1%	-0.6%	-0.9%	-1.1%	-1.4%
Simulation 2: 5% Increase in feed back coefficient with effect on state equations	4290.3	4515.9	4723.6	4885.6	5030.1	4987.9	5206.2	5250.9	5198.9	5160.4
Dynamic multipliers simulation 2	0.0%	0.0%	0.0%	0.0%	0.0%	-3.7%	-2.0%	-2.8%	-3.2%	-3.8%
Net multiplier effect of the Lucas proposition	0.0%	0.0%	0.0%	0.0%	0.0%	-2.7%	-1.3%	-1.9%	-2.1%	-2.4%

Figure 1: Actual, predicted and residual Money Rule

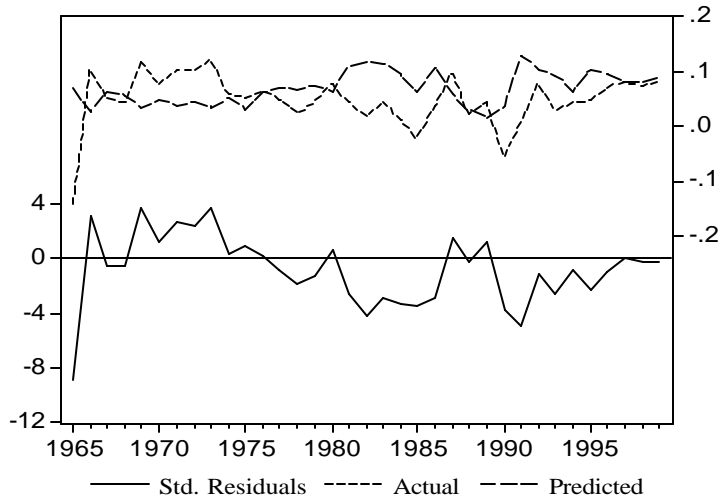
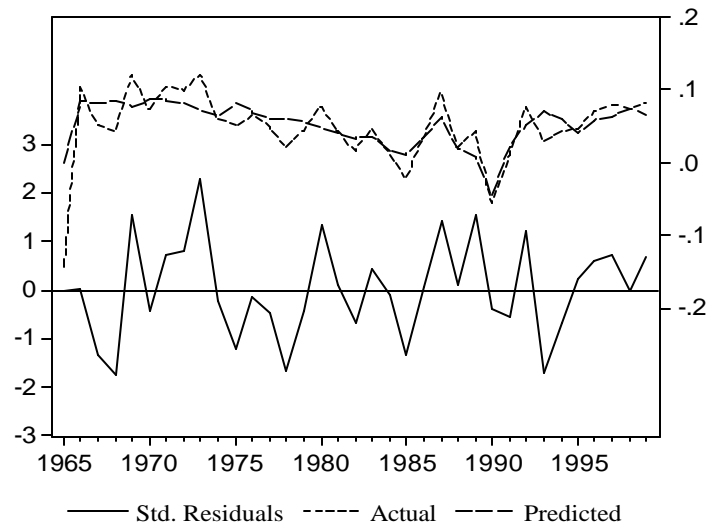
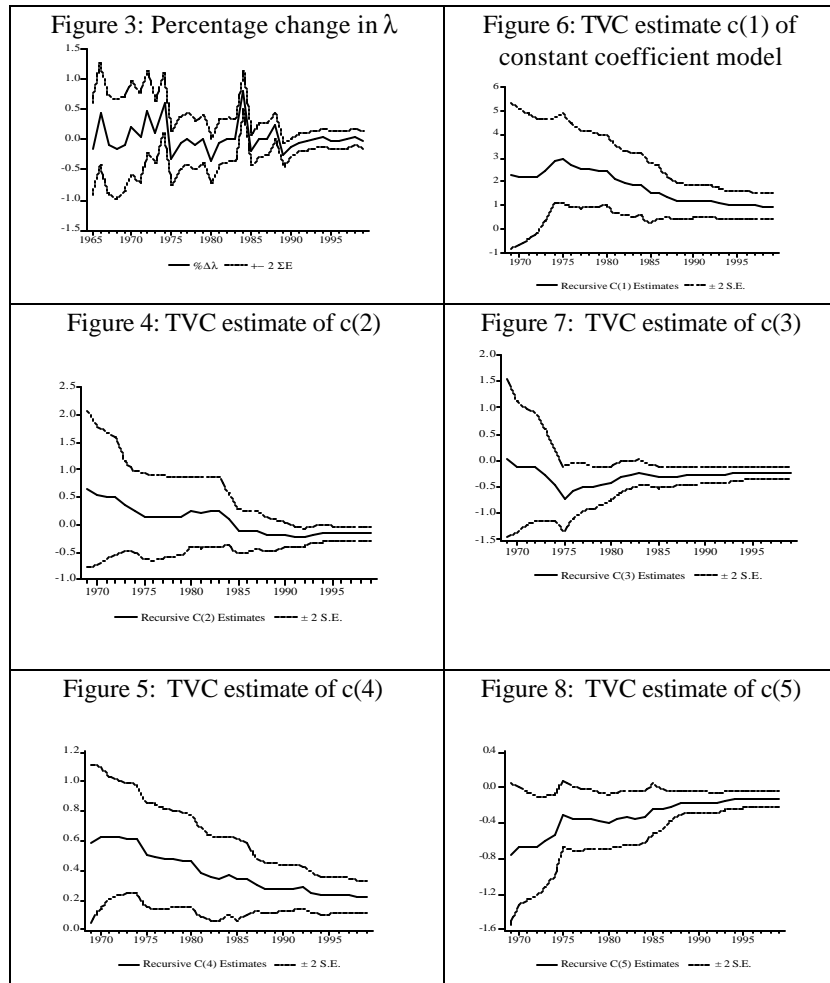


Figure 2: Actual, predicted and residual for Supply Equation



Time-varying coefficients (TVC) of money rule and supply equations



forecast and simulations (1995-1999)

Figure 9 :
Base line

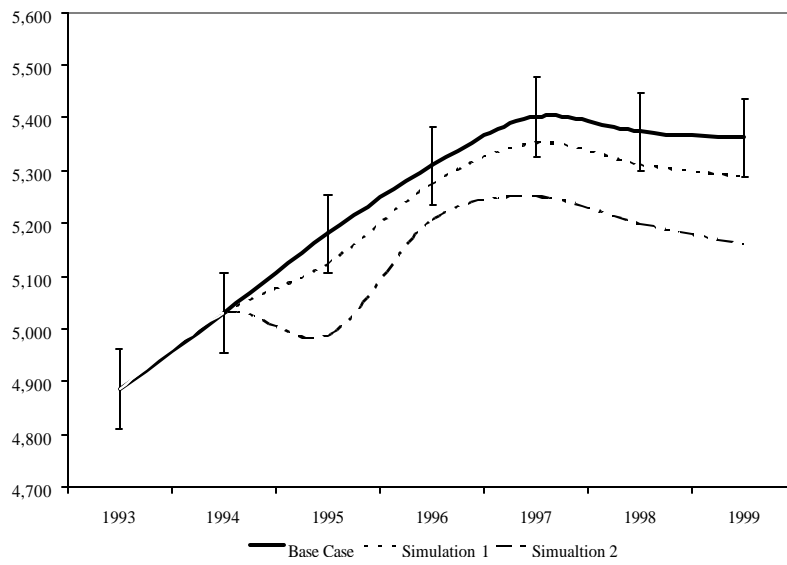
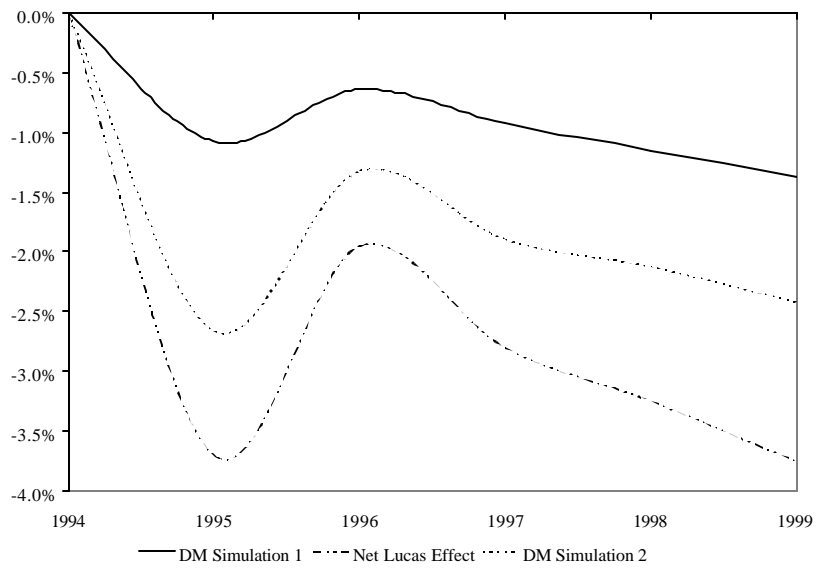


Figure 10: Dynamic Multipliers (1995-1999)



DATA DESCRIPTION, SOURCE AND CONSTRUCTION

All data were taken from the International Monetary Fund's *International Financial Statistics*, various volumes. The nomenclature is as follows:

- M Money stock M1. IMF *line* 33
- P CPI of the DR, (1995=100). IMF *line* 64
- Y Real GDP (1995=100). IMF *line* 99b