# Macroeconomics and Networks<sup>\*</sup>

THIS DRAFT: JUNE 30, 2014

Guillermo Lagarda<sup>†</sup>

### INTER AMERICAN DEVELOPMENT BANK

### Abstract

With the aim of measuring the extent to which main the macroeconomic and the financial sectors are exposed to shocks that may cause systemic failures, this paper presents a topological network analysis using cross-sector borrowing and lending. In particular, sectorial interconnectedness is explored using metrics of centrality and connectivity. The economy is configured into two networks, one at a macro level and a second one at a banking sector level. Given that the initial architecture of the network, the optimal allocations of funds are found through a multiagent general equilibrium model and an iterative procedure where the intervention of a Lender of Last Resort (LLR) avoids a collapse. The role of the LLR is mimicked by reconfiguring the whole network such that a more homogenous risk sharing improves the resilience of the overall system. In order to verify this, the network structure is shocked through the banking system and by using the strength of the links the simulation quantifies a domino-like effect throughtout the network. The findings show that, besides being useful to identify potential pitfalls in the interconnectedness of sectors, networks are convenient to test how reconfiguring the links would provide resilence to the overall system. In particular, a more symmetric and dense configuration in combination with lower centrality , renders a more resilient system and therefore milder effects on the macroeconomic variables.

### 1 Introduction

Network analysis has been recognized as an interesting methodological tool for characterizing complex interactions between agents. By modeling these economic interactions, network analysis may better explain certain economic phenomena. Additionally, an important trait of networks is that they allow to model interconnections that otherwise would be difficult to model with standard approaches.<sup>1</sup> A network approach to macro financial systems is particu-

<sup>\*</sup>This Paper should not be reported as representing the views of the IADB. The views expressed in this Paper are those of the author and do not necessarily represent those of the IADB or IADB policy. This is a Preliiminary Draft, please do not cite.

<sup>&</sup>lt;sup>†</sup>Inter American Development Bank and Boston University

 $<sup>^{1}</sup>$  These include VARs or macroeconomic general equilibrium models.

larly important for assessing stability and can be instrumental in capturing the externalities that the risk associated with a single agent may create for the entire system. A better understanding of network externalities may facilitate the adoption of a macro-prudential framework for supervision or improved targeted regulation.

Networks have been in economists' minds for a while now, albeit not precisely applied to systemic risk. Recent bibliography, especially during and after the Great Recession's turmoil, focused on modeling interactions between economic agents and assessing the resilience of financial systems towards systemic risk. Following this trend, policy makers and central bankers have increasingly grown awareness about financial interconnectedness. While many of the main interactions within the financial system agents are somehow known, a better understanding of them is becoming an important concern in macro-financial surveillance.

Moreover, network techniques may have a larger scope to assess systemic risk at different levels. Resembling a pyramid, network complexity may span from the most aggregate level in the economy to the roots that predetermine each stage of aggregation. At the aggregate level, it has been widely explored how the macroeconomic accounts are linked through the traditional system of national accounts, fiscal accounting, balance of payments, as well as central banks and financial system surveys. Nonetheless, each of the participant agents may have particular relations with some other parties through the credit channel, namely borrowing and lending. These deeper linkages may carry important information about the sensitivity and degree of exposure that different sectors of the economy may have to particular shocks. In this regard, network techniques have been used, among others, to describe the global configuration of international financial flows, to analyze financial contagion, and to examine the dynamics of payment systems and interbank markets.

In spite of the broadening in research regarding the nature and causes of systemic risk, there is not yet a unified view on how to better approach them. This in turn has also reflected in the large spread of views, to some extent conflicting, between how the connectedness between sectors and contagion are related. In other words, the role that learning from the topology of a network has in providing meaningful insights to untangle the drivers of contagion have not reached a uniform perspective. For instance, early economic literature in the subject, as those from Freixas, Parigi, and Rochet (2000) or the seminal work by Allen and Gale (2000) find that a more homogenous distribution of interbank claims improves the resilience of the system to the insolvency of any individual bank. Furthermore, Allen and Gale, show that a more densely interconnected financial network, risk sharing significantly reduces among members of the web since more creditors are able to absorb the losses of a shock. In contrast to this view, however, others have found that more dense networks may function as negatively as well triggering acute systemic failures. Vivier-Lirimont (2006) and Blume et al. share the view where as the number of a bank's links grow, the likelihood of a systemic collapse increases.

More generally, network analysis is useful to address two types of issues: the

effect of the network structure and the process of network formation. While the first type of question captures aspects related to overall efficiency, the second type highlights the tension between socially desirable outcomes and the outcomes that arise as a result of the self-interested action of individuals. Network theories build from two methodological approaches. The first approach draws from an overlapping literature of physics, sociology and biology. These methodologies are highly mechanical in the sense that they are based on topological definitions. The second one builds on the network economics literature, taking a micro perspective that considers how an agent's behavior is driven by incentives. Which of the two approaches is more appropriate to model financial networks depends on whether financial institutions are assumed to behave strategically or not. A microfounded (game theoretical) analysis requires that agents need to be aware of the shape of the network they belong to and the impact of the network on their gains. A mechanical approach that draws from the physics and mathematics literatures can only provide cause-and-effect insights.

In view of the existing different perspectives, this paper presents a simple framework for studying the role of the financial system architecture and macroeconomic sectors network in the system's resilience toward events that may lead to systemic failures. The model focuses primarily on the relationship between two layers of networks (the macroeconomic sector and the financial networks) and secondly, the extent of contagion through domino-like effects. Unlike most of the literature, the model allows for reconfiguring the architecture of the network by capitalizing up to n additional financial institutions. The macroeconomic layer is then characterized optimally using a three period general equilibrium model. In the initial date, banking sector borrows funds from households, whom in turn may or may not borrow from the banks. Moreover, the non-financial productive private sector perceives a liquidity constraint to operate and has to borrow from banks. All financial transactions occur on the following period, and by the end of the second period all contracts are expire. The optimal allocation at this level determines the level of liquidity of the financial sector network where interbank links are possible. The asset-liability structure that emerges from such interbank links determines the financial network, capturing the pairwise relationships between different institutions. Thus, a bank whose current assets suffer a certain hair cut may have to liquidate its liabilities. When a hair cut occurs, the "speed" at which the members of the network are informed is proportional to the strength of the link between the shocked institution and the counterparts. Depending on the structure of the financial network, this may rigger a cascade of failures: the default of a bank on its debt may lead to financial distress of its creditor banks, which in turn may default on their own counterparties, and so on. These dynamics are then replicated under the presence of a Lender of Last Resort (LLR) who decides what is better, either to raise funds to save the system or capitalize n identical institutions that will lend and borrow to the other members of the financial system.

The paper explores the properties of the main macroeconomic sectors, firstly, by means of topological network analysis. Using bilateral data on lending and borrowing captured from the Honduran Other Depository Survey (ODS), the Central Bank Survey (CBS), balance of payments, and fiscal accounts, it describes the topology of the network using different metrics of interconnectedness (such as country centrality and network density) and assess its resilience to shocks.

Given the complexity that networks carry, real data is used to visualize at each step of the document some of the metrics and results, instead of presenting in two different chapters methodology and results. The document is, therefore, structured as follows: the next chapter briefly describes the related literature. Chapter 3 briefly introduces the reader to the main features of a network. It first describes what and how the links are computed, then defines two common topological metrics, and present some graphical illustration of current networks (for the case of Honduras). Chapter 4 presents simulations to test the resilience that the current network has to respond to a shock in the banking sector. Next it develops a simple model to characterize the linkages between macro sectors and the banking institutions. The chapter ends by seeking for a reconfiguration of the network using an iterative procedure which minimizes the bail out amount by the LLR in case of a shock. Lastly, Chapter 5 present conclusions and some areas of future research.

# 2 Literature Review

A recent but growing literature focusing on the role of the architecture of the financial system as an amplification mechanism has it roots on Allen and Gale (2000) and Freixas, Parigi, and Rochet (2000). These two provided some of the first formal models of contagion over financial networks. Allen and Gale make use of an extended version of Diamond and Dybvig (1983). They show that the interbank relations that emerge to pool group shocks may induce vulnerabilities in response to shocks when they are not anticipated. Although with a different objective as this paper, Shin (2008) uses balance sheets as network linkages. He finds that securitization enables credit expansion through higher leverage of the financial system as a whole, which generates an externality where lending standards lower and increases the overall fragility.

On the nature of the assets and liabilities as drivers of contagion, Allen, Babus, and Carletti (2012) show that the pattern of asset commonalities between different banks carries hidden information that each counterpart is able to recognize. This potentially triggers contagion and hence, the likelihood of a systemic crisis. Also related is Castiglionesi, Feriozzi, and Lorenzoni (2010), who show that a higher degree of financial integration, implies more shared information that leads to more stable interbank interest rates in normal times and to larger interest rate spikes during crises.

More recently, works of Elliott, Golub, and Jackson (2013) and Cabrales, Gottardi, and Vega-Redondo (2013) study the broad question of propagation of shocks in a network of firms with financial interdependencies. The core of them is a contagion mechanism based on how holdings of different agents' assets may lead to contagion and failures. Unlike them, this paper implicitly assumes that once you are in the network, every link conducts information represented by the interbank borrowing and lending. Therefore, when a shock occurs, every agent perceives a pulse that triggers a sequence of claims on assets to cover existent liabilities and a cascade of failures.

Finally, Babus (2009) studies a model in which banks form linkages in order to insure against the risk of contagion. Not unlike Babus, this document finds that banks may insure themselves by lending and borrowing to each other. However, an important difference resides in how the insurance mechanism is derived. In particular, Babus does not allow for a free designed network. In summary, none of the above papers provide a comprehensive analysis of the relationship between the topology of the networks, systemic failures due to contagion, and the possibility of reconfiguring the network to improve resilience to shocks.

# 3 Networks

Two different uses for networks are implemented through this document. The first one seeks to identify how the main macroeconomic sectors are linked within the economy by means of a network representation. Important flow of funds are regularly moving from one sector to another through lending and borrowing. The extent these transactions expose a particular sector to shocks depends on how large are the flows between the affected sector and all its counterparts. The network representation therefore, shows a map of the current interconnectedness, and provides a quantification of the degree of centrality or relevance of each sector within the system. Moreover, the second use of networks aims to quantify losses after a simulated shock under the current network and an hypothetical reconfigured net. Reconfiguring the network is equivalent to analyzing changes in regulation or in institutional policies, and therefore it may be useful to test how resilient the new nets are.

# 3.1 The Links

In order to characterize the macro level networks several market clearing conditions have to be implemented. Lending and Borrowing within the economy should add up according to the amount of resources that are available. For instance, changes on Government Financing,  $\Delta D_t^G$ , must be covered by domestic ( $\Delta DD_t^G$ ) and foreign funds ( $\Delta FD_t^G$ ). For the lenders, these funds are accounted as asset positions (domestic from financial system  $DA_t$ , and non financial private sector,  $A_t^{H \to G}$  as well as foreign  $A_t^{NR \to G}$ ), thus

$$\Delta D_t^G = \Delta D D_t^G + F D_t^G$$
$$\Delta F D_t^G = A_t^{NR \to G}$$
$$\Delta D D_t^G = D A_t + A_t^{H \to G}$$

Similarly, the financial sector carries domestic assets by means of lending to the Government and offering credit to the private sector. It may also be the case that the financial system is lending and borrowing to/from non residents. For all these assets we might expect a liability position, domestic or foreign as well.<sup>2</sup> In general, it is possible to distinguish to some extent, the proceedings of some of these accounts. In particular, the network analysis will use information from the Depository Corporations (OSD) and Other Financial Corporations (OSF) surveys. Thus, the total foreign assets of the financial system, excluding those from the Central Bank,

$$FA_t = FA_t^{OSD} + FA_t^{OSF}$$

and similarly domestic assets of the financial system may also be divided between those coming from OSD and OSF:

$$DA_t = DA_t^{OSD} + DA_t^{OSF}$$

The liability side has an analogue relation: Foreign liabilities of the financial system will find no resident lenders and domestic liabilities local counterparts to ensure that

$$FL_t = FL_t^{OSD} + FL_t^{OSF}$$
$$DL_t = DL_t^{OSD} DL_t^{OSF}$$

whenever

$$FL_t = A_t^{NR \to B}$$
$$DL_t = A_t^{H \to B}$$

where foreign assets from non residents to financial system is denoted by  $A_t^{NR\to B}$  and domestic asset from non financial private sector  $A_t^{H\to B}$ . In general, flows depicted by NR and H flows are financing through bond markets or simply buying commercial paper.<sup>3</sup>

The liability side of the OSD and OSF may be matched to the private sector too, for instance foreign liabilities will match the asset side of the Non Residents. In analogy, domestic liabilities would be matched with household assets.<sup>4</sup>

$$FL_t^{OSD} = A_t^{NR}$$
$$DL_t^{OSD} = A_t^{HH}$$

We may find some other interrelations which are accounted to meet the sectorial consistency as Figure 1 summarizes.<sup>5</sup> The data available for Honduras was selected to implement the contents of this document. The data was extracted from the Central Bank of Honduras, however the calculations and selective aggregation of data was used to simplify the calculations.

 $<sup>^{2}</sup>$  Equity is not considered in the formation of the networks.

<sup>&</sup>lt;sup>3</sup> Deposits are excluded.

 $<sup>^4</sup>$  In fact, domestic liabilities of the OSD my have OSF as counter part or Central Bank, too. Here it is assumed that the central Bank enters only as an exogenous player, but OSF are indeed accounted.

<sup>&</sup>lt;sup>5</sup> The consistency rules follow the IMF manual of Financial Programming.



Fig. 1: Interrelation across sectors of the Ecomomy

### 3.2 Metrics

For each year where data is available an adjacent matrix, (A) is formed using cross sector flow of funds registered in matrix (M). The first step is to quantify topological measures to identify how central or how connected a particular node is. The document will focus in two simple measures: Centrality (node degree) quantifies how many times a particular node is used to move funds from one sector to another. Density measures simply illustrate the likelihood of connectedness between sectors (nodes).

#### Country centrality

Node degree: Counts the number of connections (links) for each sector (node). Since the aim is to work with a directed network, we have in-coming links for borrowers and out-going links for lenders. Therefore, it is computed the out-degree (the number of outgoing links) for each sector by counting the sectors to which it lends (its debtors) and in-degree (the number of incoming

links) counting the sectors from which it borrows (its creditors), as follows:<sup>6</sup>

$$DO(i) = A_i \cdot 1$$
$$DI(i) = A'_i \cdot 1$$

where i depicts the row of the matrix.

#### Network density

Connectivity. Network connectivity is the number of links that exist between sectors (or total node degree) expressed as a share of the total possible number of links. It represents the likelihood of connection between two sectors in the system. Let  $a_t$  be the observed number of links (corresponding to positive flows  $a_{tij} > 0$ ) in the matrix  $A_t$ . With n lenders and n borrowers in the system network, the degree of connectivity is given by

$$DC_t = \frac{a_t}{n_t^2}$$

Figure 2 illustrate the value of these measures for the macroeconomic level network. Not surprisingly centrality has increased, in particular after 2009. This corresponds to better market conditions and the foreign aid that Honduras received during these periods. Density turns to be more informative: The size of the flows between sectors increased, improving connectivity. In particular, an important spike occured between 2010 and 2012, consistent with better market conditions and market confidence regarding political stability.

### 3.3 Representations

The following figures show the macro networks for 2002 and 2012. Each node corresponds to a particular sector. The design of the network defines how intense is the relation between sectors (width of the links) and if there is a bilateral flow of funds (arrows). Figure 3 depicts the macro networks in two topological styles. The first element of the panel corresponds to the configuration of the network in 2002. It is shaped as a polygon as every sector has the same importance for this display. These types of webs weight equally each sector (node), and allow us to visualize the concentration or density of each. For instance, 2002 and 2012 (lower graph) show basically the same structure, and certainly that should be the case, as the intersectorial consistency should hold mostly at all times.

<sup>&</sup>lt;sup>6</sup> Some other topological measures of centrality are node strength: Is the total value of flows originating or terminating in a given node. In our case, in-strength for sector i (NSI) is the total amount of cross-sector credit it receives, whereas out-strength for sector i (NSO) is the total amount of cross-sector credit it lends. Out-strength and in-strength are computed by substituting matrix A for matrix M in the node degree formulas presented above. Node strength is the simplest weighted network indicator that captures the intensity of financial relationships among sectors. Relative node strength: Focuses on the relative importance of lenders as providers of financial capital, and respectively, that of borrowers as destinations for financial investment in the network. Borrowerj's dependence on lender i is the share of inflows it receives from i in her total borrowing. Hence, relative node out-strength increases with the lender's relative importance.



Fig. 2: Upper Left: Out-Degree Centrality. Upper Rigth: In-Degree Centrality. Lower: Density (connectivity)

The strenght of the links for 2012 are depicted on the upper right panel. Some sectors interact heavily and therefore they have larger centrality. The shape of the net also describes how central the node, that is, a node far away from the others implies fewer interaction.

The aggregate macro network gives us some reasonable insights of how sensitive each node might be. However, this will not give us further knowledge of how the actual lending and borrowing might affect the network. In order to address this, the banking sector must be included as a second layer in the network. To do so, a second excercise takes into account the number of banks in the economy, (I), and their weight on overall banking activity. Because the overall banking assets and liabilities must add up the values found in the OSD, the second layer should either replace the OSD node or represent an independent network linked through the OSD node. For simplicity, this excercise will follow the former rather than the latter. To construct the new links, the banks in the system are ranked according to their relative size of assets and liabilities within the whole system. Uniformly they supply funds to the other sectors and the excess resources are distributed within the interbank links. Loosely speaking we have:

$$FA_t^{OSD} = \sum_i \beta^{i \to S} + e_t$$

Because we use a uniform assignment of funds, then

$$\beta = \frac{FA_t^{OSD}}{I}$$

However, it may be the case that for some, this value is larger than their own



Network Representations 2002 and 2012: Honduras

Fig. 3: Upper Left Panel: Macro Sector Network in 2002. Upper Right Panel: Macro Sector Network 2009. Lower Panel: Macro Sector Network 2012

assets, therefore the following rule is implemented

$$BA_t^{i \to S} = \min(\frac{FA_t^{OSD}}{I}, BA_t^i)$$

The remaining amount,  $(e_t)$ , is then assigned within the interbank network. The procedure again uses the same iterative rule of uniformly assigning funds until all are exhausted. Figure 4 shows the weighted networks once we replace the OSD with banks and its multidimensional representation. The interconnectedness is complex but we may identify several nodes (sectors or banks) that have more links. For instance, following the Upper Panel, OSF corresponds to node #2 while node 3 and 4 account for the public sector. Bank 12 (node #18) seems to be the largest fund provider while bank 10 (node #16) the least interconnected.

Once we allow the desing of the network to account for the strenght and interconnectedness between nodes (lower panel), we may observe that Bank 12 and 16 are centered and with much more connections. Bank 9 and 10 are further away from the core as we should expect from institutions with lower connectedness. The network also shows connection of some nodes to itself: This is only because in order to match the value of assets with liabilities the equity has to be considered as a balance that may be available.

### 4 Simulation and Reconfiguring

Now that we have visualized and to some extent quantified or measured centrality and density, this next section will seek to test how a shock in a particular node would spread across the network. In order to do so, a failure is characterized as the banks inability to back up its liabilities. Once this happens, the shortage on the asset side of the counterpart will not be able to cover its own liabilities, and this will reduce the asset side of all its counter parts, this cycle repeats at every node. Whenever a node's assets are reduced to zero, then all its links vanish. The exercise tries to replicate a scenario where an unhealthy balance sheet firstly reduces the asset value of a particular institution. When this happens, it is impossible to liquidate some of them to cover liabilities and comply with contract and borrowing agreements. This event causes every agent who lended or had a financing agreement with the failing institution to find a reduction on their available assets, which in turn will not be enough to convert their own liabilities. This domino effect is allowed to occur with no further intervention, and the exercise counts the number of steps before the financial sector simply stops operating (destruction of links). To simulate this, one may think of the adjacent matrix as a transition system. Since each flow represents the monetary amount that is exposed between two sectors, then the probability of spreading the illiquidity to two different nodes would vary according to the amount exposed between each other. Therefore, the adjacent matrix is normalized in such a way that the liability positions that a particular sector has with everyone else adds to one. Therefore, each element of the matrix assigns a share of total borrowing (or lending) to each institution where there is an



Extended Network Representation 2012: Honduras

Fig. 4: Upper Panel: Extended Network including comercial banks. Lower Panel: Extended Network including commercial banks (free designed)

active link. With no further assumptions involving priority partners or term contracts, the normalized matrix could be thought as a transition probability matrix. This new matrix is then used to simulate multiple stories after a shock hits a particular node. At each stage or propagation, the overall asset/liability balance is verified. A sector or bank continues operating while it has assets, eventhough it may be the case that fewer assets creates a larger imbalance. This would simulate a case where the institutions have to liquidate all their existing assets before stopping their operations. Implicitly, this assumption is posing the hypothetical event where the liquidation of assets does not cause a bank run.<sup>7</sup> Once a particular node runs out of assets then it fails to function and breaks a link. This exercise is repeated by shocking different sectors and test the propagation of the shock. Figure 3 shows the results of this simulation process. The upper panel depicts the shock on Bank #17 (node #23), which is one of the most interconnected. The shock is designed, to reduce its initial assets on 10%. This shock, given the adjacent matrix, propagates through the complete network mainly within the largest counterparts at the banking layer and by impacting the OSF (node #2) at the macro layer. The simulation shows that after, on average, 14 claims on Bank 17's liabilities, the institution leaves the system. However, this is not the first institution to collapse. Bank #7 and Bank #16 leave the network much before Bank #17. This is due to the composition of their current balance sheets and the size of the initial claims that other institutions had on them. However, when the shock impacts initially a smaller institution then the propagation effects are milder. Figure 5 lower panel depicts the effects of this simulation. In this scenario, the shock is imputed on Bank  $\#10 \pmod{\#16}$ , interestingly no financial institution collapses. Indeed, the same bank is able to stabilize, albeit with lower balance sheet value. The graph also shows that the effects of the shock are somehow more homogenous across nodes, compared to the first simulation. These results demonstrate how the effects in a network might not be linear and to some extent seem to imply that centrality matters when assessing for propagation and systemic failure.

# 4.1 The Lender of Last Resort

To explore further the usefulness of a network analysis, a lender of last-resort (LLR) facility is introduced to compensate for the lack of liquidity. The iteration after the shock remains the same, except for the fact that the lender of last resort provides resources until the system stabilizes again. The size of the funds required to stabilized the system would replicate the magnitud of a rescue package. The lender of last resort, however, will only intervene when a financial institution suffers twice of an asset liquidation that is not able to fully cover the claims on its liabilities. That is, whenever the pulse of the shock hits back the node where it started, and the bank is unable to honor any percentage of the claimed amount, then the LLR will cover the liquidity requirements. This

 $<sup>^7</sup>$  Another way to think about this, is that everything happens within a single period. All the propagating effects happen from the moment the institution was shocked until the banking hours end or the bank collapses.



Frequency of Claims Received by Each Agent in the Network

Fig. 5: The Y axis denote relative frequency and X axis the number of the node. Upper: The initial shock impacted B17 which has one of the most connections. Lower: Initial Shock through B10 which is one of the least interconnected.

may happen plenty of times, as a very interconnected node may propagate the shock to, say, 4 other nodes which in turn may extend the shock to some other arms. This procedure might cause at least some of the already harmed nodes to receive a second round of asset liquidations. This iteration will stop whenever the system stabilizes again, or it fully collapses. The detailed procedure is as follows:

- First step, select a specific bank to be shocked or be given an arbitrary chosen assets haircut. This haircut may be thought in two ways: 1) from one day to the next, the bank will not have enough money to cover the liabilities, in fact it will only be capable to cover up to the value of the assets after the shock. 2) Conversely, this means that a counterpart did not honor his/her liabilities with the chosen bank.
- The realization of a haircut will trigger an immediate reaction from all lenders. In fact, all lenders will claim their assets on the affected institution. However, which lender claims first is given through a random draw from the network probability distribution matrix. This implies that the larger the loans given from a particular institution to the affected bank, the higher probability it has to be the first to claim its assets. Moreover, if chosen, it will demand the totality of the funds lent to the shocked bank.
- If the bank's assets are enough to cover the initial claim it goes ahead and honors the debt. However, a second counterpart, from the remaining lenders is randomly selected through the probability matrix: again knocks on the door and makes a claim on the totality of the funds lent to the shocked bank.
- This iteration continues until the shocked bank cannot cover all of its liabilities- *a default*.
- Once this happens, the lending bank sees a haircut on its own assets, consequently triggering, its debtors to demand their funds back (randomly following the probability matrix). This includes those banks that defaulted (have no assets) but they may still have claims on other banks that have been affected too.
- Whenever, banking institutions see that their assets diminish to zero and their claims on other banks have zero value, they cease to function, breaking all of its links. This is repeated until the system stabilizes itself or it completely collapses.

When does a LLR come into play? An LLR makes an attempt to save the system by intervening the moment that the first bank cannot meet its debts obligations. However, the LLR will only cover those claims once they ran out completely of assets. That is, a LLR will only intervene the moment it realizes that there will be no other agents in the network that can recapitalize (claims on other banks) the banks that see their assets diminish due to triggers.



### Frequency of Claims Received by Each Agent in the Network

Fig. 6: The Y axis denote relative frequency and X axis the number of the node. Lender of Last Resort: At every needed stage the LLR intervene causing the spike on #23.

Figure 6 shows the results of the simulation with a lender of last resort. The simulation mimicks the aforementioned version for Bank #17. With the exception that Bank #17 is replaced by the needed size of a lender of last resort to stabilize the overall system (position #23 in Figure 6). The prescence of a LLR subtantially alleviates the impact of the negative shock. The incidence becomes strongly homogenous across institutions and sectors, improving the risk sharing capacity overall. Most of the benefits are absorbed by the largest banking institutions, while the midsize ones remain with a relative high exposure. The macro layer of the network is the least benefited, as they do not receive a direct flow from the LLR in terms of funding. Moreover, this outocme to some extent supports both, the literature results on connectivity and risk sharing, but also the importance of counting with access to a LLR. Given the composition of the adjacent matrices and the methodology explained, a LLR makes the system substantially more resilient to shocks. Unfortunaley, the scope of this network analysis is not enough to provide a comprehensive strategy to create or form funds equivalente to a LLR facility.<sup>8</sup>

<sup>&</sup>lt;sup>8</sup> The IMF in some countries has implemented special credit facilities to respond to these kind of shocks. However, to gain access to these instruments many monteray and fiscal arrangements may be needed.

# 4.2 Reconfiguring

An important question that arises when any network analysis is used to study propagation is linked to how and why a particular network is formed. While literature normally makes use of participation constraints and a game theoretical approaches to explain how each link is established, from the policy point of view these stragies may not be fully useful. The most standarized conclusions about networks imply that a better connected system will create a risk sharing system that would evenly spread the shock and would not allow itself to fail. From the point of view of the policy maker, or if we want to think about a policy planner, the market incentives should be modified to induce a better network. Moreover, this planner is also concerned with its own participation as a lender of last resort within the economy. For instance, the fiscal sector would be willing to intervene if its fiscal and debt sustainability are not at risk. Similarly, the Central Bank, may be willing to act if its actions would not affect other targets or its own monetary position. To characterize a novel approach to address these interesting features, the document models a multiagent economy where households have preferences over consumption goods and financial products. Furthermore, firms require of intermediation to function and the banking system operates by borrowing from agents to later lend to other financial and non financial agents. The household sector is caracterized as follows

$$\max_{C_1, C_2, B, D} U(C_1) + V(B_2, D_2) + \beta U(C_2)$$
  
s.t.  
$$D_1 + C_1 = Y_1 + B_2$$
  
$$C_2 = Y_2 - RB_1 + D_1 R^f$$

where B is the borrowing from the banking system and D lending to the banking system. Moreover, the non financial private sector operates with a labor technology such that

$$\max_{h_1, L_2} f(h_1) - wh_1 + L_2 - RL_1$$
s.t.
$$wh_1 \le L_2$$

where h stands for the labor demand, L the borrowing from the banking system.<sup>9</sup> Basically, the firms require credit for working capital; in this two period modelling L in first period is given. The overall banking system is characterized by a single decision agent. They choose elements of their balance sheets to maximize their intermediation profits:

$$\max_{L,B,\mathbb{R}} R[L+B] - R^f D$$

 $<sup>^9</sup>$  Conversely, the private sector may be merged into a housohold-firm agent and deal with only two optimization problems. The results ought not vary since they are equivalent.

s.t.  

$$\mathbb{R} + L + B + \phi(B, L) = D$$

$$\rho D \le \mathbb{R}$$

where  $\mathbb{R}$  are required reserves on liabilities and  $\phi(\cdot)$  denotes the technology of intermediation. Finally, the model is closed by implementing the familiar market clearance conditions.

Moreover, the behavior of the interbank system (the second layer of the network) is inspired in similar fashion as Allen and Gale (2000). However, the model here goes further and departs from their framework by first taking as given (and using) the current topology of the network (before and after a desired shock). Given the amount needed to bail the system, the LLR must decide what is better, either to just raise the funds to help the system or capitalize n identical institutions that will lead and borrow to the other members of the financial system. Therefore, the objective is to first optimize the macro level variables. Second, using the optimization conditions as constrains, simulate a shock in the financial institution layer and seek to reconfigure a network that would replicate or maybe improve the current resilience. In other words, the key decision is how central each node should remain in the second layer. For simplicity and as before it is assumed that the size of the shock on the assets of an institution implies a direct fail from the counterparts to cover a fraction of their liabilities, or conversely the incapacity to honor all its debts. Figure 7 illustrates the logic behind the network structure that is being assumed. Step by step the reconfiguring procedure is as follows:

- The first layer network has to comply with market clearing conditions and consistent macro accounting that is incorporated into the multisector equilibrium. Optimality at this level implies that the links between every two sectors must comply with the optimality conditions related to them. Moreover, solving the system gives the equilibrium allocation for total borrowing and lending in the economy (OSD).
- The set of all financial institutions that form part of the OSD in equilibirum, must match the assets and liabilities of the OSD. If this is not the case, the hypothetical LLR secures the discrepancies by either holding or transfering them uniformly across banks. The LLR keeps the net value of the total discrepancies.
- Once borrowing and lending are in equilibrium in both the macro and banking system, a shock is intruduced in the same fashion as in the previous chapter. Thus, the second layer, which represents the interbank connections is shocked (a single bank) and the LLR reacts to maintain the macro level equilibrium conditions. That is, given the OSD macro equilibrium, the LLR transfers, as explained in chapter 3.1, the amount required to avoid the collapse of the system.

• Thereafter, this LLR is faced with a choice between introducing up to n new banking institutions or simply collecting by any other means the amount of the liquidity demand. If  $\hat{M}_j(n)$  represents the network matrix with n financial institutions at iteration j, F being the liquidity requirement to stabilize the system, then the LLR whill choose n such that given an identical shock  $s^{10}$ 

$$F_{j+1} = \min_{n} \{F(\hat{M}_{j}(n), s), F(s)\}$$

$$s.t$$

$$\sum_{i=0}^{I+n(j)} \hat{M}_{j}^{i}(n) \cdot 1 = F(s)$$

$$m_{j}^{i \to k}(n) = \frac{F}{nK}$$

where  $i \in I$  represents the banking institution,  $k \in K$  the counterpart (K nodes), and  $m_j^{i \to k}(n)$  stands for the i, k element of  $\hat{M}_j(n)$ . Thus, at each iteration, the new configuration is subject to the same shock (s), obtaining an update on the funds required to compensate the shortage of liquidity  $F_{j+1}$ .

- After the first update, the credit shortage acts as a shock to the first layer (macro layer), and generates a transition path until it reaches back the previous macro equilibrium.<sup>11</sup>
- The iteration is repeated until the system over all stabilizes:  $\mid F_{j+1}-F_{j}\mid <\epsilon.$

Solving this procedure may be analytically complex and therefore the computational method is the best way to approach it. Figure 8 shows the results of this procedure. The upper graph corresponds to the reconfigured network after after 5 iterations and the lower graph after 7. No more iterations were needed as the next reconfiguration was marginally better than the previous. The procedure rendered results in line with Allen and Gale (2000), as the incidence after the shock reduces substantially when the interconnections approach to a complete network. This is actually a sign of a larger risk sharing, but most importantly to a larger and homogenous risk sharing. The size of the bail that a hypothetical LLR would have to apply reduces sharply. In fact, the incidence of the LLR moves from 26 percent (Figure 6) of the banking activity to about 15 percent and 11 percent, with 22 and 24 banks, respectively (Figure 9). Moreover, the

 $<sup>^{10}</sup>$  Recall that here a shock is a story of events randomly drawed from the network's probability matrix.

<sup>&</sup>lt;sup>11</sup> The model is simplified by using a two period model. Therefore, at macro level there are only two values for the involved variables. At every reconfiguring of the network, occurs a deviation from the equilibrium, however this is restored by the end of the second period since the model parameters are assume to remain fixed.



Fig. 7: The two layers of networks linked by the OSD and the existance of an hypothetical LLR

topological metrics reveal an important result: Node centrality (relative the new number of nodes) reduces on average for all the members of the network (about 20% less than the original configuration. In contrast, density increases sharply and above 0.8 (recall that macro density in 2012 was about 0.44). The overall behavior of the new architecture also responds to a more symmetric composition, although, the LLR could also be requested to allocate not uniformly funds across the network.

# 5 Conclusions

The potential usefulness of network techniques in analyzing interconnections between different sectors of the economy has not been yet extensively explored. However, networks in the context of systemic risk have taken center-stage in academic and policy debates in the aftermath of the 2008-09 global crisis. Moreover, not much is known about the structural properties and time-evolution of the network of cross-sector linkages, which are key to understanding how a domestic financial system reacts to shocks, and whether and where systemic risk may emerge. Nonetheless, while there is significant research focused on characterizing financial networks, a benchmark framework where financial networks interact with macroeconomic sectors is yet absent. This paper aimed to cover that gap by analyzing a comprehensive network linking the banking institutions and the main macroeconomic sectors. In order to do so, the main macro accounting interrelations are established consistently across sectors. At the same time, the banking institutions are characterized through the asset and liability positions the have with all other agents of the economy. Thus, the channel that connects the macro sectors and the banks are borrowing and lending. In order to illustrate objectives of the document a topological approach is used to analyze the main features of the current networks. The main findings show that both, density and centrality have increased, especially after 2009. Furthermore, to what extent the current configuration of the net is resilient was tested by a simulation given the implicit probability matrix. In absence of a lender of last resort, when a shock hits a banking institution with high centrality the

a simulation given the implicit probability matrix. In absence of a lender of last resort, when a shock hits a banking institution with high centrality the propagation is so strong that the overall banking system collapses. In contrast, when a low centrality bank is shocked, the system is able to stabilize, albeit, with lesser value of balance sheets. However, if a lender of last resort exists, the negative incidence on all members of the network reduces significantly. In fact, on average the lender of last resort intervenes 26 percent of all transactions. Unfortunately, this may translate into large amount of resources that might or might not be available for the economy. This led to questioning could a better configuration of the network improve risk sharing and minimize the size of a bail out? To address this, a macro multi-agent model is combined with an iterative procedure to find an optimal number of members for the network. The iterative procedure rendered results in line with some theoretical results. For instance, given the implicit probability matrix of the network, a more symmetric and better-connected matrix resulted in smaller bail out amounts and smaller credit crunches. In other words, the optimal reconfiguring renders lower levels of centrality for all members of the network. Moreover, the incidence after a shock reduces significantly from an average of 6% to 3%, and interestingly seems to lead to a more homogenous risk sharing. The lender of last resort is required only 11 percent of the times, less than half than the current configuration. This results show that given the assumptions a better structure of the intersectorial network will make the economy more resilient to sudden shocks. Reducing centrality and increasing density require of important changes on the main incentives that currently exist. Improved regulation and monitoring are important but not sufficient to ensure resilience. The figure of a lender of last resort poses a second complication. The construction of the model assumes the availability of funds, when actual resources are most likely scarce or conditional. In any case, both represent challenges, that if addressed seem to strengthen the overall resilience of the economy. A number of questions emerge from the analvsis. While it has been established that cross-sector lending is a key channel of transmission of financial crises, how the topology of financial networks relate to the emergence of systemic risk remains underexplored. How do the properties of the different networks-banking, FDI, trade, and remittances-compare and how do countries' degrees of connectedness interact in different webs of relationships? What is the empirical relationship between connectedness and the way in which shocks get amplified or diffused? These and related questions remain interesting avenues to explore in future work.

# 6 References

1. Acemoglu, Daron, Vasco M. Carvalho, Asuman Ozdaglar, and Alireza Tahbaz-Salehi (2012), "The network origins of aggregate fluctuations." Econometrica, 80, 1977-2016.

- Acemoglu, Daron, Asuman Ozdaglar, and Alireza Tahbaz-Salehi (2010), "Cascades in networks and aggregate volatility." NBER Working Paper 16516.
- Allen, Franklin and Ana Babus (2009), "Networks in finance." In The Network Challenge: Strategy, Profit, and Risk in an Interlinked World (Paul R. Kleindorfer and Yoram (Jerry) Wind, eds.), 367–382, Wharton School Publishing.
- Allen, Franklin, Ana Babus, and Elena Carletti (2012), "Asset commonality, debt maturity and sys- temic risk." Journal of Financial Economics, 104, 519–534.
- Allen, Franklin and Douglas Gale (2000), "Financial contagion." Journal of Political Economy, 108, 1–33.
- Ambrus, Attila, Markus Mobius, and Adam Szeidl (2012), "Consumption risk-sharing in social networks." NBER Working Paper 15719.
- Babus, Ana (2009), "The formation of financial networks." Discussion Paper 06-093, Tinbergen Institute.
- Battiston, Stefano, Domenico Delli Gatti, Mauro Gallegati, Bruce Greenwald, and Joseph E. Stiglitz (2012), "Liaisons dangereuses: Increasing connectivity, risk sharing, and systemic risk." Journal of Economic Dynamics and Control, 36, 1121–1141.
- 9. Berman, Abraham and Robert J. Plemmons (1979), Nonnegative matrices in the mathematical sciences. Academic Press, New York.
- Billio, Monica, Mila Getmansky, Andrew W. Lo, and Loriana Pelizzon (2012), "Econometric measures of connectedness and systemic risk in the finance and insurance sectors." Journal of Financial Economics, 104, 535–559.
- 11. Blume, Larry, David Easley, Jon Kleinberg, Robert Kleinberg, and E´va Tardos (2011), "Network for- mation in the presence of contagious risk." Proceedings of the 12th ACM Conference on Electronic Commerce.
- Brunnermeier, Markus K. and Martin Oehmke (2012), "Bubbles, financial crises, and systemic risk." In Handbook of the Economics of Finance (George M. Constantinides, Milton Harris, and Rene´ M. Stulz, eds.), volume 2, North Holland.
- 13. Brunnermeier, Markus K. and Lasse Heje Pedersen (2005), "Predatory trading." The Journal of Finance, 60, 1825–1863. Caballero, Ricardo J. and Alp Simsek (forthcoming), "Fire sales in a model of complexity." Journal of Finance.

- 14. Cabrales, Antonio, Piero Gottardi, and Fernando Vega-Redondo (2013), "Risk-sharing and contagion in networks." Working Paper.
- 15. Castiglionesi, Fabio, Fabio Feriozzi, and Guido Lorenzoni (2010), "Financial integration and liquidity crises." Working paper.
- Cifuentes, Rodrigo, Gianluigi Ferrucci, and Hyun Song Shin (2005), "Liquidity risk and contagion." Journal of the European Economic Association, 3, 556–566.
- 17. Cohen-Cole, Ethan, Eleonora Patacchini, and Yves Zenou (2013), "Systemic risk and network formation in the interbank market." Working Paper.
- 18. Cormen, Thomas H., Charles E. Leiserson, Ronald L. Rivest, and Clifford Stein (2001), Introduction to Algorithms. MIT Press, Cambridge, MA.
- Dasgupta, Amil (2004), "Financial contagion through capital connections: A model of the origin and spread of bank panics." Journal of the European Economic Association, 2, 1049–1084.
- 20. Diamond, Douglas W. and Philip H. Dybvig (1983), "Bank runs, deposit insurance, and liquidity." Journal of Political Economy, 91, 401–419.
- 21. Diamond, Peter A. (1982), "Aggregate demand management in search equilibrium." Journal of Political Economy, 90, 881–894.
- 22. Eboli, Mario (2012), "A flow network analysis of direct balance-sheet contagion in financial net- works." Working paper.
- Eisenberg, Larry and Thomas H. Noe (2001), "Systemic risk in financial systems." Management Science, 47, 236–249.
- 24. Elliott, Matthew, Benjamin Golub, and Matthew O. Jackson (2013), "Financial networks and contagion." Working Paper. Financial Crisis Inquiry Commission (2011), "The Financial Crisis Inquiry Report: Final Report of the National Commission on the Causes of the Financial and Economic Crisis in the United States." U.S. Government Printing Office, Washington, DC.
- 25. Ford, Lester Randolph and Delbert Ray Fulkerson (1956), "Maximal flow through a network." Canadian Journal of Mathematics, 8, 399–404.
- Freixas, Xavier, Bruno M. Parigi, and Jean-Charles Rochet (2000), "Systemic risk, interbank relations, and liquidity provision by the central bank." Journal of Money, Credit and Banking, 32, 611–638.
- 27. Gorton, Gary and Andrew Metrick (2011), "Securitized banking and the run on repo." Journal of Financial Economics (forthcoming).

- Gorton, Gary B. (2010), Slapped by the Invisible Hand: The Panic of 2007. Oxford University Press, USA.
- Haldane, Andrew G. (2009), "Rethinking the financial network." Speech delivered at the Financial Student Association in Amsterdam, The Netherlands, April 2009. http://www.bankofengland.co.uk/publications/ Documents/speeches/2009/speech386.pdf.
- Holmstrom, Bengtand JeanTirole(1998), "Private and public supply of liquidity." Journal of Political Economy, 106, 1–40.
- Jackson, Matthew O. and Asher Wolinsky (1996), "A strategic model of social and economic net- works." Journal of Economic Theory, 71, 44 – 74.
- 32. Kiyotaki, Nobuhiro and John Moore (1997), "Credit chains." Working Paper.
- Krishnamurthy, Arvind (2010), "Amplification mechanisms in liquidity crises." American Economic Journal: Macroeconomics, 2, 1–30.
- Leitner, Yaron (2005), "Financial networks: Contagion, commitment, and private sector bailouts." The Journal of Finance, 60, 2925–2953.
- 35. Longstaff, Francis A. (2010), "The subprime credit crisis and contagion in financial markets." Journal of Financial Economics, 97, 436 450.
- Lorenzoni, Guido (2008), "Inefficient credit booms." The Review of Economic Studies, 75, 809–833.
- 37. Plosser, Charles I. (2009), "Redesigning financial system regulation." A speech at the New York University Conference "Restoring Financial Stability: How to Repair a Failed System".
- Rotemberg, Julio J. (2011), "Minimal settlement assets in economies with interconnected financial obligations." Journal of Money, Credit and Banking, 43, 81–108.
- 39. Shin, Hyun Song (2008), "Risk and liquidity in a system context." Journal of Financial Intermediation, 17, 315 329.
- Shin, Hyun Song (2009), "Securitisation and financial stability." The Economic Journal, 119, 309–332.
- Shleifer, Andrei and Robert W. Vishny (1992), "Liquidation values and debt capacity: A market equi- librium approach." The Journal of Finance, 47, 1343–1366.
- 42. Sorkin, Andrew Ross (2009), Too Big to Fail: The Inside Story of How Wall Street and Washington Fought to Save the Financial System—and Themselves. Viking, New York.

- 43. Stulz, Rene´ M. (2010), "Credit default swaps and the credit crisis." Journal of Economic Perspectives, 24, 73–92.
- 44. Vivier-Lirimont, Se´bastian (2006), "Contagion in interbank debt networks." Working Paper.



Fig. 8: Upper Panel: The leftmost network reconfiguration denotes an increase from 17 banks to 22. The rightmost corresponds to the case where the number of banks increase from 22 to 24. The weighted representation depicts a larger number of interconnections and much more symmetry as the number of banks increase. Lower Panel: Similarly, the leftomost corresponds to the stage where the net moved from 17 to 22 banks. The lower rightmost corresponds to the final iteration with 30 banks. The graphs show how more regular is the network in the second case, due to a larger symmetry.



Frequency of Claims Received by Each Agent in the Network

Fig. 9: The Y axis denote relative frequency and X axis the number of the node. Upper: Incidence after shock when 22 banks are in the network. Lower: Incidence after shock when 24 banks are in the network.